

## REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

AD-A258 003



ted to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this burden to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Office of Management and Budget: Paperwork Reduction Project (0704-0188), Washington, DC 20503.

2

RT DATE

3. REPORT TYPE AND DATES COVERED  
FINAL 1 Jun 91 - 31 Jul 92

## 4. TITLE AND SUBTITLE

"COMBINATORIAL RELIABILITY &amp; REPAIR" (U)

## 5. FUNDING NUMBERS

61102F

2304ES

## 6. AUTHOR(S)

Professor Lorenzo Traldi

## 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Lafayette College  
219 Markle Hall  
Easton PA 18042-8. PERFORMING ORGANIZATION  
REPORT NUMBER

AFOSR-91-

## 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

AFOSR/NM  
Bldg 410  
Bolling AFB DC 20332-644810. SPONSORING/MONITORING  
AGENCY REPORT NUMBER

AFOSR-91-0274

## 11. SUPPLEMENTARY NOTES

DTIC  
ELECTE  
NOV 25 1992  
S E D

## 12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release;  
Distribution unlimited

## 12b. DISTRIBUTION CODE

UL

## 13. ABSTRACT (Maximum 200 words)

During the Summers of 1991 and 1992 the principal investigator research the use of combinatorial techniques in analyzing problems involving the repair of K-terminal networks, which are networks given with a distinguished subset K of the vertex-set. (Ordinary graph theory can be viewed as being equivalent to the special case K=V(G).) There are large bodies of existing literature describing the uses of such techniques in analyzing (a) ordinary networks (for which the reliability and repair problems are very similar to each other, being connected with certain matroids associated to the network) and (b) K-terminal reliability problems: the fundamental problem is to find out how these two bodies of material generalize to the repair of K-terminal networks. The original proposal listed several specific areas of research, of which the second and fifth turned out to be the most interesting.

## 14. SUBJECT TERMS

## 15. NUMBER OF PAGES

4

## 16. PRICE CODE

17. SECURITY CLASSIFICATION  
OF REPORT

UNCLASSIFIED

18. SECURITY CLASSIFICATION  
OF THIS PAGE

UNCLASSIFIED

19. SECURITY CLASSIFICATION  
OF ABSTRACT

UNCLASSIFIED

## 20. LIMITATION OF ABSTRACT

SAR

LAFAYETTE COLLEGE  
DR LORENZO TRALDI

DTIC QUALITY INSPECTED

Combinatorial Reliability and Repair  
AFOSR-91-0274  
final technical report

Accession For	
NTIS	CRA&I
DTIC	TAB
Unannounced	
Justification	
By _____	
Distribution /	
Availability Codes	
Dist	Avail and / or Special
A-1	

## 1 Summary

During the Summers of 1991 and 1992 the principal investigator researched the use of combinatorial techniques in analyzing problems involving the repair of  $K$ -terminal networks, which are networks given with a distinguished subset  $K$  of the vertex-set. (Ordinary graph theory can be viewed as being equivalent to the special case  $K = V(G)$ .) There are large bodies of existing literature describing the uses of such techniques in analyzing a) ordinary networks (for which the reliability and repair problems are very similar to each other, being connected with certain matroids associated to the network) and b)  $K$ -terminal reliability problems; the fundamental problem is to find out how these two bodies of material generalize to the repair of  $K$ -terminal networks. The original proposal listed several specific areas of research, of which the second and fifth turned out to be the most interesting; they will be discussed last.

The first area mentioned in the proposal involved the use of edge-packing techniques to produce approximations to reliability measures associated with the repair of networks. The principal investigator made little progress in this area, not because edge-packing techniques do not generalize to network repair problems but because their generalization seems so obviously valid as to require little investigation.

The third area mentioned in the proposal involved the use of imperfect vertices and faces in the analysis of networks, particularly in conjunction with the star-delta and delta-star transformations. This area differed from the first

92-30226

5  
2

in that the idea of using imperfect faces seems not to have appeared in the literature of network reliability before; it has the effect of making it possible to have exact star-delta and delta-star equivalences (they are only approximate, otherwise). The specific problem mentioned in the proposal, formulating versions of the series and parallel reductions that would be compatible with the use of imperfect faces, was solved soon after the proposal was written, and the first version of the paper *On the star-delta transformation in network reliability* discussing these ideas was written in the Summer of 1990. Recently the paper was revised somewhat and presented at the Seventh Quadrennial International Conference on Graph Theory, Combinatorics, Algorithms and Applications in Kalamazoo, Michigan, on June 5, 1992.

The fourth area mentioned in the proposal involved the polygon-to-chain reductions of Satyanarayana and Wood [7]. Like the edge-packing techniques mentioned earlier, these reductions obviously generalize from reliability problems to repair problems, and the principal investigator has not found anything else to say about them.

The sixth area involved the generalization of techniques from undirected networks to directed ones. This generalization is often a difficult one, for many reasons. For instance, "connectedness" in undirected graphs is associated with a partition of the vertex-set, consistent in simple ways with the partitions encountered when an edge is deleted or contracted, whereas "reachability" in directed graphs is not associated with any such simple partition. Connected with this is the fact that the principal investigator has not been able to find an immediate generalization of his work with undirected networks; investigating the existence and extent of such a generalization remains an interesting problem.

The second area mentioned in the original proposal involved the notion of reliability domination introduced by Satyanarayana and Chang [5] (for undirected  $K$ -terminal networks) and refined by Satyanarayana and others in the last ten years or so. In particular, Satyanarayana and Tindell introduced a notion of  $(K, j)$ -domination in their study of a  $K$ -terminal version of the chromatic polynomial [6], and it turns out that this  $(K, j)$ -domination has many properties very strongly reminiscent of Satyanarayana's and Chang's original theory, e.g., the networks of  $K$ -terminal domination 0 and  $\pm 1$  can be characterized, and these characterizations can be used to fine-tune the implementation of deletion/contraction algorithms to calculate reliability measures associated with network repair. This is discussed in the paper *Reliability*

*domination, generalized activities, and the repair of K-terminal networks*, written in 1991.

The fifth area mentioned in the paper involved the rather general issue of relating notions associated with network repair to other combinatorial notions. Two very important such relationships that are in the literature are the relationship between activities and interval partitions of (portions of) the power-set of the edge-set of a network (cf. [1] for a discussion) and the relationship between reliability domination and the Crapo  $\beta$ -invariant [3, 4]; both of these relationships turn out to involve matroid theory. This is the area the principal investigator has worked in most energetically since the Summer of 1991. Two papers have already been written, *Generalized activities and K-terminal reliability. II* about using the principal investigator's *K*-terminal generalized activities to construct interval partitions of the power-sets of the edge-sets of *K*-terminal networks, and *Crapo's  $\beta$ -invariant and K-terminal networks* about one way of generalizing the  $\beta$ -invariant to *K*-terminal networks.

Since finishing the second of these in July of 1992, the principal investigator has continued to look into the use of matroids in analyzing *K*-terminal networks, particularly the relationship between the  $\beta$ -invariant and notions of reliability domination. He has found that there is a family of matroids  $M_j$  associated to a *K*-terminal network, with  $\beta(M_j)$  equal to the  $(K, j)$ -domination;  $M_1$  is of the type introduced by Huseby [3, 4]. These matroids can be used to give another viewpoint on the results of *Reliability domination, generalized activities, and the repair of K-terminal networks* regarding the  $(K, j)$ -dominations. The construction of the  $M_j$  can also be generalized to a construction of a "polygon matroid" associated to any hypergraph, with a set of hyperedges being considered independent if it is possible to choose one edge within each hyperedge, so that the resulting set of edges is independent in the usual sense.

In the near future the principal investigator will continue to investigate the relationship between matroid theory and reliability and repair problems associated with *K*-terminal networks. After clarifying the relationship between the  $M_j$  and the  $(K, j)$ -dominations by trying to use the  $M_j$  to re-prove some of the theorems of *Reliability domination, generalized activities, and the repair of K-terminal networks*, and revising the paper accordingly, he will consider other questions about the structure of these matroids; for instance, is it possible to prove a version of Whitney's 2-isomorphism theorem that

will give sufficient and necessary conditions for two  $K$ -terminal networks (or more generally, two hypergraphs) to have isomorphic associated matroids? He also hopes to formulate a purely matroidal theory generalizing  $K$ -terminal reliability and repair problems, with "ports" (distinguished elements) of a matroid playing a role analogous to that of the set  $K$  of distinguished vertices of a  $K$ -terminal network; such a generalization should ideally provide a common context in which the similarities and differences between reliability and repair problems, and between  $K$ -terminal and ordinary problems, could be studied. Among other things, it should provide a context unifying the partitions of *Generalized activities and  $K$ -terminal reliability. II* with the activities-based partitions usually associated with ordinary ( $K = V(G)$ ) graphs. He also intends to look into the properties of the polynomial that gives the expected number of needed repairs of a  $K$ -terminal network; this polynomial seems similar in many ways to the usual reliability polynomial, and has the " $K$ -terminal  $\beta$ -invariant" of *Crapo's  $\beta$ -invariant and  $K$ -terminal networks* as its leading coefficient.

## 1.1 References for the Summary

- [1] C. J. Colbourn, *The combinatorics of network reliability*, Oxford Univ. Press, Oxford, 1987.
- [2] G. Gordon and L. Traldi, *Generalized activities and the Tutte polynomial*, Discrete Math. **85** (1990), 167-176.
- [3] A. B. Huseby, *A unified theory of domination and signed domination with applications to exact reliability computations*, Statistical Research Report, Institute of Mathematics, University of Oslo, Oslo, Norway, 1984.
- [4] A. B. Huseby, *Domination theory and the Crapo  $\beta$ -invariant*, Networks **19** (1989), 135-149.
- [5] A. Satyanarayana and M. K. Chang, *Network reliability and the factoring theorem*, Networks **13** (1983), 107-120.
- [6] A. Satyanarayana and R. Tindell, *Chromatic polynomials and network reliability*, Discrete Math. **67** (1987), 57-79.
- [7] A. Satyanarayana and R. K. Wood, *A linear-time algorithm for computing  $K$ -terminal reliability in series-parallel networks*, SIAM J. Comput. **14** (1985), 818-832.
- [8] L. Traldi, *Generalized activities and  $K$ -terminal reliability*, Discrete Math. **96** (1991), 131-149.